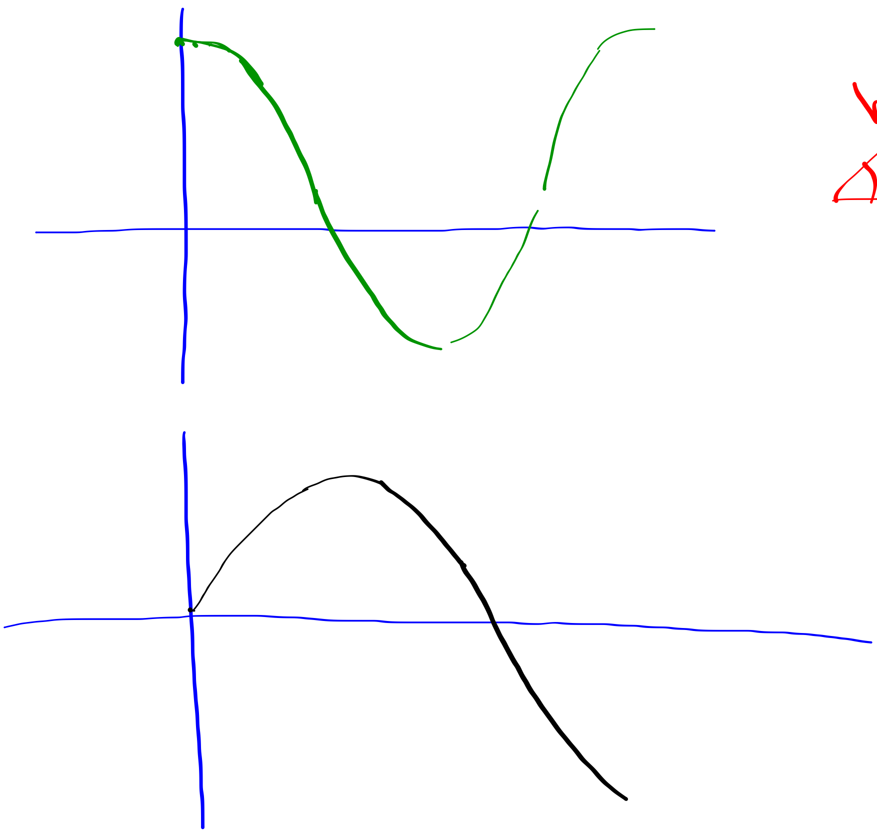
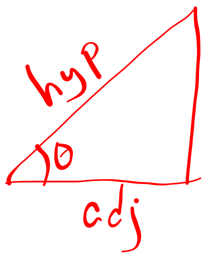
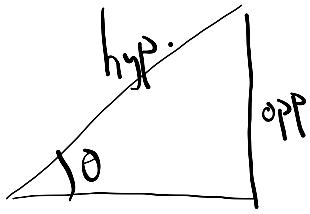
Cos x related to x-coordinate on a circle

day $2^3 \cdot 7$

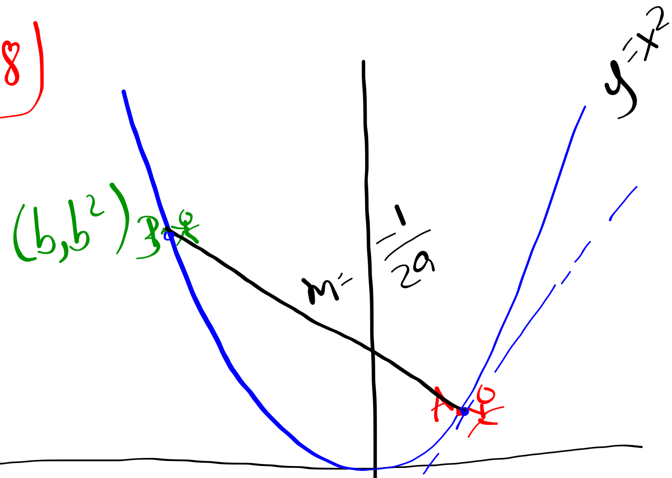






4.1/78

day $2^3 \cdot 7$



a) $m_{tangent}$ at $A=(a,a^2)$

$y=x^2$

$\frac{dy}{dx}=2x \quad \left.\frac{dy}{dx}\right|_{x=a}=2a$

$m_{\perp}=\frac{-1}{2a}$

b) eqn of connector

$y-a^2=\frac{-1}{2a}(x-a)$

$y=a^2-\frac{1}{2a}(x-a)$

c) what are the coordinates of B? (b,b^2)

$x^2=a^2-\frac{x-a}{2a}=\left(a^2+\frac{1}{2}\right)-\frac{x}{2a}$

$x^2+\frac{x}{2a}-\left(a^2+\frac{1}{2}\right)=0$

$x=\frac{-\frac{1}{2a}\pm\sqrt{\frac{1}{4a^2}+4\left(a^2+\frac{1}{2}\right)}}{2}=\frac{-1}{a}\pm\sqrt{\frac{1}{16a^2}+\left(a^2+\frac{1}{2}\right)}$

$=\frac{-1}{a}\pm\frac{1}{|a|}\sqrt{\frac{1}{16}+a^4+\frac{1}{2}a^2}$

$=-\frac{1}{a}\pm\frac{1}{a}\sqrt{\left(a^2+\frac{1}{4}\right)^2}$

$\downarrow$

$\frac{-\frac{1}{2a}}{2}\pm\frac{1}{2}\sqrt{\frac{1}{4a^2}+4\left(a^2+\frac{1}{2}\right)}$

$\frac{-\frac{1}{2a}}{2}\pm\frac{1}{2}\sqrt{4}\sqrt{\frac{1}{16a^2}+\left(a^2+\frac{1}{2}\right)}$

day $2^3 \cdot 7$

connector

$y - a^2 = \frac{-1}{2a}(x-a)$ $y = x^2$

$y = a^2 - \frac{1}{2a}(x-a)$

$$\frac{b^2 - a^2}{(b-a)} = \frac{-1}{2a}\frac{(b-a)}{(b-a)}$$

$$b + a = \frac{-1}{2a}$$

$$b = \frac{-1}{2a} - a$$

$$b = \frac{-1-2a^2}{2a}$$

78d)

day $2^3 \cdot 7$

$b = \frac{-1-2a^2}{2a}$

$d^2 = (b-a)^2 + (b^2 - a^2)^2$

$d^2 = \left(\frac{-1-2a^2}{2a} - a\right)^2 + \left(\left(\frac{-1-2a^2}{2a}\right)^2 - a^2\right)^2$

$= \left(\frac{-1-4a^2}{2a}\right)^2 + \left(\frac{1+4a^2+4a^4}{4a^2} - \frac{4a^4}{4a^2}\right)^2$

$\checkmark\ d^2 = \left(\frac{-1-4a^2}{2a}\right)^2 + \left(\frac{1+4a^2}{4a^2}\right)^2$

$\checkmark\ d^2 = \frac{1+8a^2+16a^4}{4a^2} + \frac{1+8a^2+16a^4}{(4a^2)^2}$

e)

$f(a) = \frac{1}{4}a^{-2} + 2 + 4a^2 + \frac{1}{16}a^{-4} + \frac{1}{2}a^{-2} + 1$

represents d^2

$f'(a) = -\frac{1}{2}a^{-3} + 8a - \frac{1}{4}a^{-5} - a^{-3}$

$f'(a) = 8a - \frac{3}{2}a^{-3} - \frac{1}{4}a^{-5} = 0$

$8a^6 - \frac{3}{2}a^2 - \frac{1}{4} = 0$

$32a^6 - 6a^2 - 1 = 0$

day $2^3 \cdot 7$

4.2/30) $f(x) = x^2\sqrt{9-x^2}$ $(-3,3)$

f^n increasing or decreasing?

$$f'(x) = 2x\sqrt{9-x^2} + x^2\left(\frac{1}{2\sqrt{9-x^2}}(-2x)\right)$$

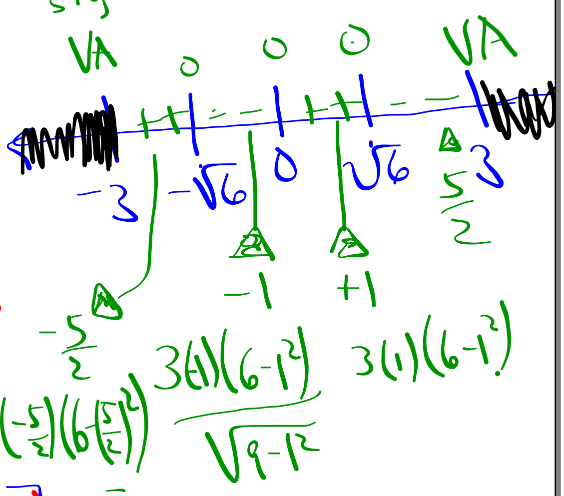
$$= 2x\sqrt{9-x^2} - \frac{x^3}{\sqrt{9-x^2}}$$

$$f'(x) = \frac{2x(9-x^2)}{\sqrt{9-x^2}} - \frac{x^3}{\sqrt{9-x^2}} = \frac{18x\underbrace{-2x^3-x^3}_{-3x^3}}{\sqrt{9-x^2}}$$

$$\boxed{f'(x) = \frac{3x(6-x^2)}{\sqrt{9-x^2}}}$$

crit pts
Ⓐ $f'(x) = 0$
Ⓑ $f'(x)$ und

sign chart of first deriv



crit pts

Ⓐ $18x(1-3x^2) = 0$

$\Rightarrow x = 0, -\sqrt{6}, \sqrt{6}$

B $9-x^2=0 \Rightarrow x = -3, +3$

$f(x)$ is increasing on

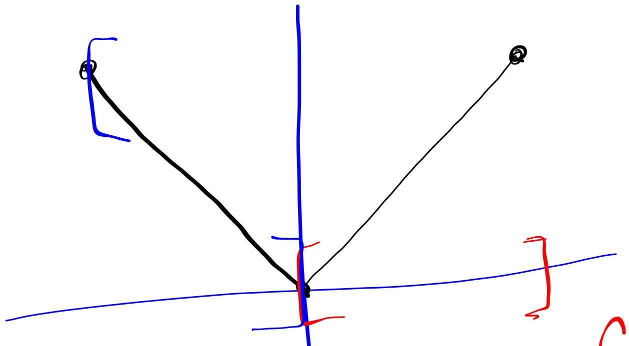
$(-3, -\sqrt{6}] \cup [0, \sqrt{6}]$

$f(x)$ is decreasing on

$[-\sqrt{6}, 0] \cup [\sqrt{6}, 3)$

$f(x) = |x| \quad [-2, 2]$

day $2^3 \cdot 7$



Remember

$f(x)$ is increasing on an interval if for every $x_1 < x_2$, $f(x_1) < f(x_2)$.

$f(x)$ is increasing depends on:

* every pair of points IN AN INTERVAL

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day $2^3 \cdot 7$

$f(x) = x^3 + 4x$

inc vs dec.

$f'(x) = 3x^2 + 4$

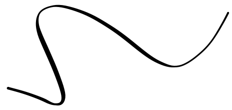
$3x^2 + 4 = 0$

$3x^2 = -4$

$x^2 = -\frac{4}{3}$

$x = \pm\sqrt{-\frac{4}{3}}$ $\emptyset$

No Solution



CRIT PTS

A) $f'(x) = 0$

NO SOLUTIONS

B) $f'(x)$ undefined

ALWAY def.

NO CRIT PTS

factoring (or careful thought) ... the key to success

day $2^3 \cdot 7$

$e^{-x} - xe^{-x} = 0$

$e^{-x}(1-x) = 0$

$\emptyset \qquad x = 1$

$e^{-x} = xe^{-x}$

$\frac{e^{-x}}{e^{-x}} \quad \frac{xe^{-x}}{e^{-x}}$

$1 = x$

Any x makes this 0?

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